**ESE 605-001: Modern Convex Optimization** **Spring 2021**

# Homework 4

*Assigned: 03/16/2021* *Due: 03/26/2021*

## Homework must be LaTeX'd or it will not be graded.

You are expected to provide complete and rigorous solutions to all questions. Do not leave portions of your solutions as "exercises for the grader." If you use external resources please be sure to cite them.

**Problems from Boyd & Vandenberghe:** 5.12, 5.14, 5.17, 5.29, 5.31

**Problems from additional exercises**
`https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook_extra_exercises.pdf` : A4.1, A4.4

**Problems from Boyd & Vandenberghe:** 6.3, 6.6(a-c)

**Bonus questions:** Bonus questions are completely optional. If a certain threshold of correctness is exceeded, you will earn up to an additional 2 marks per question on your assignment grade. These are fun, challenging problems, and we ask that you try your best to get as far into the proofs/answers as you can *without consulting any outside sources!* Once you get stuck, indicate the point at which you were stuck in your solutions with a "**I made it this far on my own**," after which, you should indicate what outside source you consulted in order to finish the problem, in accordance with the Penn Academic Integrity policy.

1. (2 marks) B&V 5.44

2. (3 marks) Consider the following quadratic program

$$\begin{array}{ll} \text{maximize} & x^\top M_0 x \\ \text{subject to} & x^\top M_k x \geq b_k,\ k = 1, \ldots, K, \end{array} \tag{1}$$

   where each $M_i$, $i = 0, \ldots, K$ is a symmetric *Metzler* matrix, i.e. a matrix with nonnegative off-diagonal elements. Show, using duality or other arguments, that the solution to this problem can be found by solving the following semidefinite program

$$\begin{array}{ll} \text{maximize} & \operatorname{trace}(M_0 X) \\ \text{subject to} & \operatorname{trace}(M_k X) \geq b_k,\ \ k = 1, \ldots, K \\ & X \succeq 0. \end{array} \tag{2}$$

3. (3 marks) In B&V 4.45 on the last homework, we saw how semidefinite programming could be used to approximately solve polynomial optimization problems such as the one posed in equation (1) through *sum-of-squares optimization.* Formulate optimization problem (1) as a sum-of-squares optimization problem using the monomial basis $f(x) = [1; x]$, and then using the result of the previous bonus problem, show that this sum-of-squares optimization problem is *exact*, i.e., that it exactly solves the original polynomial optimization problem. *Hint: looking at the dual to problem* (2) *may be useful here.*