

Homework 6

Assigned: 03/22/2020

Due: 04/06/2020

Homework must be \LaTeX 'd or it will not be graded.

Problems from Boyd & Vandenberghe: 5.1, 5.3, 5.5, 5.12, 5.17, 5.39, 5.43

Problems from additional exercises

https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook_extra_exercises.pdf: A4.3

Extra problem bonus questions: These bonus questions are completely optional. These are problems that would have normally been assigned as homework problems, but are now being added as additional problems for those of you who would like to attempt them. An extra 0.5 marks can be earned by successfully completing them.

1. Problems 5.42 and 5.44 from B&V
2. Problems A4.5 and A4.6 from additional exercises

“Standard” bonus questions: These are the previous standard bonus questions, and are again completely optional. If a certain threshold of correctness is exceeded, you will earn an additional 0.5 marks on your assignment grade. These are fun, challenging problems, and we ask that you try your best to get as far into the proofs/answers as you can *without consulting outside sources!* Once you get stuck, indicate the point at which you were stuck in your solutions with a **“I made it this far on my own,”** after which, you should indicate what outside source you consulted in order to finish the problem, in accordance with the Penn Academic Integrity policy.

1. Consider the following quadratic program

$$\begin{aligned} & \text{maximize} && x^\top M_0 x \\ & \text{subject to} && x^\top M_k x \geq b_k, \quad k = 1, \dots, K, \end{aligned}$$

where each M_i , $i = 0, \dots, K$ is a symmetric *Metzler* matrix, i.e. a matrix with nonnegative off-diagonal elements. Show, using duality or other arguments, that the solution to this problem can be found by solving the following semidefinite program

$$\begin{aligned} & \text{maximize} && \text{trace}(M_0 X) \\ & \text{subject to} && \text{trace}(M_k X) \geq b_k, \quad k = 1, \dots, K \\ & && X \succeq 0. \end{aligned}$$