ESE 605-001: Modern Convex Optimization **Spring 2020**

Homework 2

Assigned: 01/28/2020 *Due: 02/07/2020*

Problems from Boyd & Vandenberghe: 3.1, 3.5, 3.14, 3.22, 3.26, 3.36
• Note that for 3.5, you may assume that $f(t)$ is differentiable.

Problems from additional exercises
`https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook_extra_exercises.pdf:` A2.1, A2.2

Bonus questions: Bonus questions are completely optional. If a certain threshold of correctness is exceeded, you will earn an additional 0.5 marks on your assignment grade. These are fun, challenging problems, and we ask that you try your best to get as far into the proofs/answers as you can *without consulting outside sources!* Once you get stuck, indicate the point at which you were stuck in your solutions with a "**I made it this far on my own**," after which, you should indicate what outside source you consulted in order to finish the problem, in accordance with the Penn Academic Integrity policy.

1. Problem 3.5 without the assumption that $f(t)$ is differentiable.
2. Show that $H(x) = \left(\sum_{i=1}^n x_i^{-1}\right)^{-1}$ is concave on $\mathbb{R}^n_{++}$.
3. Let $z = x + iy$, for i the imaginary number satisfying $i^2 = -1$. Let $f(z)$ be a bounded function defined on the strip $\{x + iy : a \leq x \leq b\}$, that is holomorphic in the interior of the strip and continuous on the whole strip. Define
$$M(x) := \sup_y |f(x + iy)|.$$
Prove that $\log M(x)$ is a convex function on $[a, b]$.